

THE RISE IN F_2^p AT HERA

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Abstract

We show that the rise in F_2^p at small x and large Q^2 seen at HERA is indeed the non-Regge double asymptotic scaling behaviour expected from the perturbative emission of strongly ordered hard gluons. An alternative explanation, in which there is no strong ordering, and a new hard Reggeon is generated, is also tried but found wanting: its theoretical short-comings are betrayed by its failure to properly account for the HERA data.

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The structure function $F_2^p(x, Q^2)$, recently measured[1] by the HERA experiments ZEUS and H1 in the hitherto unexplored region $10^{-4} \lesssim x \lesssim 10^{-2}$, $5\text{GeV}^2 \lesssim Q^2 \lesssim 10^5\text{GeV}^2$, rises dramatically both as x decreases and Q^2 increases. The form of this rise may be clearly exhibited by using the variables[2]

$$\sigma \equiv \sqrt{\ln \frac{x_0}{x} \ln \frac{t}{t_0}}, \quad \rho \equiv \sqrt{\ln \frac{x_0}{x} / \ln \frac{t}{t_0}}, \quad (1)$$

where $t \equiv \ln(Q^2/\Lambda^2)$. Rescaling F_2^p by a simple multiplicative factor $R'_F \equiv N\sigma^{1/2}\rho e^{\delta\sigma/\rho}$, where $\delta = \frac{61}{45}$, we may then plot it on a logarithmic scale against σ . The resulting plot (fig. 1) is interesting in two respects: firstly when σ is large enough all the data lie on a single line, quite independently of the value of ρ (provided ρ too is large enough), and secondly the rise of $\log R'_F F_2^p$ with σ is linear.[2] The slope of the rise[3] is 2.37 ± 0.16 (dotted line in fig. 1).

Remarkable though it is, such a rise was not unanticipated: the behaviour of F_2 at small x was computed in perturbative QCD soon after the discovery of asymptotic freedom[4], using the operator product expansion (at leading twist), the renormalization group, perturbation theory (at one loop) and assuming that at some low starting scale Q_0 the small x behaviour of F_2 is given by conventional Regge behaviour, and thus flat. The resulting perturbatively generated non-Regge behaviour takes the form

$$F_2 \sim \mathcal{N} f(\frac{\gamma}{\rho})(R'_F)^{-1} e^{2\gamma\sigma}, \quad (2)$$

where \mathcal{N} and $f \sim 1 + O(\rho^{-1})$ depend on the (soft) non-perturbative input at Q_0 . Since $2\gamma \equiv 4\sqrt{N_c/\beta_0} = 2.4$ is simply a numerical constant, perturbative QCD thus predicts precisely the double scaling behaviour[2] described above: when F_2 is rescaled by a factor $R_F \equiv e^{-2\gamma\sigma} R'_F$ to remove both the linear rise and the sub-asymptotic effects R'_F , the resulting structure function is asymptotically independent of both σ and ρ .

The rise in F_2 is thus generated perturbatively through iteration of the simple processes $g \rightarrow gg$ and $g \rightarrow \bar{q}q$: the splitting functions $P_{gg}(x)$ and $P_{qg}(x)$ are both singular at leading order as $x \rightarrow 0$. In fact at small x the gluon evolution equation reduces to a wave equation in light-cone variables $\xi \equiv \log \frac{x_0}{x}$, $\zeta \equiv \log \frac{t}{t_0}$, with a (negative) mass term which generates an exponential growth in xg inside the light-cone (see fig. 2: turn it anti-clockwise through 45°). This then drives a similar growth in F_2 . This explains why the natural variables to use when $x < x_0$ and $t > t_0$ are $\sigma = \sqrt{\xi\zeta}$ and $\rho = \sqrt{\xi/\zeta}$, and also why a generic exponential rise (σ -scaling) will be produced isotropically (ρ -scaling) whenever

the boundary conditions set at $x = x_0$ and $t = t_0$ are sufficiently soft (and in particular $g(x; t_0) \sim x^{-1}$ as $x \rightarrow 0$), so that the main source of the waves is close to the origin.

Double asymptotic scaling seems to set in just as precociously as Bjorken scaling: almost all the HERA data falls within the scaling region. For this reason we take $Q_0 = 1$ GeV; higher values would imply large double scaling violations not seen in the data. To illustrate this point we display in fig. 3 the MRS prediction[6] $D0'$, a soft distribution fitted to pre-HERA data, and then evolved (at two loops) from $Q_0^2 = 4$ GeV². Although similar in shape to the double scaling curves[2] (shown dotted in fig. 3), $D0'$ agrees less well with the HERA data. Dropping the starting scale to 1 GeV would have given a prediction which, while having the same shape as double scaling, also had the correct normalization.[5]

Note that although 1 GeV is at the boundary of the nonperturbative region, we only compare to data with $Q^2 \gtrsim 5\text{GeV}^2$, where higher twist corrections to F_2 are negligible. Higher twist evolution below this scale may be absorbed into similar uncertainties in the starting distribution.

While double scaling captures the essential physics of the rise in F_2 , there are several corrections still to be included:

- i) Sub-asymptotic corrections, both at one and two loops; these may (and have been[2][5]) computed explicitly; the dominant uncertainty is now the shape of the initial gluon distribution.
- ii) Post-asymptotic corrections, due to the leading singularities in the anomalous dimensions at higher orders in α_s ; these are now known explicitly[7] and may be included in the Altarelli-Parisi equation to all orders. In this way the the strong k_T ordering implicit at leading order is gradually relaxed in a controlled way as we go to smaller x . In the HERA region such corrections are still quite small however[8].* Subleading singularities might also be calculable.
- iii) Higher twist (screening) corrections, necessary to restore unitarity post-asymptotically; these are probably very small in the HERA region[9], though they are difficult to compute reliably.

Thus in the ‘soft pomeron’ approach (perturbative evolution of an initially soft non-perturbative input), higher order corrections are all (except possibly iii) under control, in the sense that they are small in the HERA region, and may furthermore be systematically

* Shift the axes of fig. 9 of ref.[8] by $\frac{1}{10}$ and $\frac{1}{4}$ respectively, to take into account our choice of x_0 and Q_0 : almost all the HERA data then lies within these authors’ convergence criterion.

computed within a well defined scheme. This is essentially because the operator product expansion and renormalization group are used to control the expansions. By contrast in the ‘hard pomeron’ approach (see ref.[10] for a critical review) these fundamentals are abandoned in favour of a particular diagrammatic resummation. This inevitably leads to difficulties.

The starting point for this latter approach is the ‘BFKL equation’ which resums logs of $1/x$ at fixed Q^2 ; k_T ordering is abandoned at the outset. Solution of the equation gives asymptotically (at small x and large Q^2) the Regge-like behaviour

$$F_2 \sim \mathcal{N} \sqrt{Q^2/Q_0^2} x^{-\lambda}, \quad \lambda = 12 \log 2 \frac{\alpha_s}{\pi}. \quad (3)$$

Such a rise violates double scaling[2], and indeed is much stronger than that seen at HERA (it would imply an exponential rather than linear rise in fig. 1, for example). However it need not be taken too literally, because a number of important theoretical problems remain unresolved:

I) The BFKL equation does not consider evolution in Q^2 , and thus does not resum leading logs. The equation is thus inconsistent with the renormalization group, and its solution cannot be related to known distributions at larger x . Alternative equations[11][12], which also have no k_T ordering but include Q^2 evolution, are difficult to solve, but seem to give numerical results consistent with leading order Altarelli-Parisi evolution. This is because when the coupling runs, the hard pomeron cut dissolves into a sequence of poles[13], and the right-most pole has only a small residue.

II) When the k_T ordering implicit in the conventional leading twist evolution equations is abandoned, contributions with different twist become mixed together. This leads to nonperturbative infrared effects which cannot be easily factored out, except by projecting onto the leading twist.[7] An attempt to solve this problem may be found in ref.[14]. The implementation of the (kinematically necessary) ultraviolet cutoff on k_T is also problematic.[15] Subleading corrections, while probably important (to ensure momentum conservation, for example), are as yet unknown.

III) Since unitarity is now violated more quickly since the growth (4) is much stronger than (2), screening corrections are now much larger[16]; they are also even more difficult to calculate.

While most of these problems remain unresolved, it is not yet really possible to compare the ‘hard’ approach with the data. Despite (or perhaps because of) this, it has

recently become popular to employ a ‘hybrid’ approach, in which the perturbatively based small- x behaviour (3) is used as a nonperturbative boundary condition at Q_0 , which is then evolved using the conventional two loop evolution equations. The resulting Q^2 dependence is then relatively weak, as the Lipatov cut is to the right of the leading singularities in the anomalous dimensions.[10] Since such an approach makes little sense theoretically, it is rather satisfying that the data themselves contradict it: a hard starting distribution propagates inside the light-cone anisotropically, generating strong violations of ρ -scaling.[2] This is demonstrated rather clearly by the MRS prediction[6] D' , displayed in fig. 4 (the ‘hard’ pomeron curves of ref.[2], also with $\lambda = \frac{1}{2}$, are shown dotted for comparison). In fact D' fits the data even less well than $D0'$, essentially because the sharp rise at small x is not matched by a corresponding rise at large Q^2 . Of course by reducing λ and/or using a boundary condition with only a very small relative admixture of hard to soft behaviour, it is still possible to fit the data: the recent MRS H fit mimics double scaling in precisely this way. *

In conclusion, the ‘hard’ pomeron is neither a proven consequence of perturbative QCD, nor is it as yet visible in the structure function data. Indeed, the simple prediction of perturbative QCD[4] has been beautifully confirmed[2][3] by the measured[1] rise in F_2 at HERA. This is fortunate, because it means that perturbative QCD can be used to make reliable predictions at small x . It should thus be possible to use the HERA data to further test QCD, measure α_s , and predict QCD backgrounds at the LHC by extrapolating to yet higher values of σ .

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* In fact there is only around 3% of the hard solution in MRS(H): see eqn(24) of ref.[17].

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Figure Captions

- Fig. 1. The rise in F_2^p : $R_F' F_2^p$ vs. σ . Also shown is the double scaling prediction[2], and a fitted straight line for comparison.
- Fig. 2. The x - t plane, showing the different kinematic regions, and in particular the double scaling region, with scaling co-ordinates σ and ρ .
- Fig. 3. Double scaling plots of $R_F F_2^p$ vs. a) σ and b) ρ . The data are taken from ref.[1], and the curves are those of the MRS parton distributions D0'. The double scaling curves [2] are shown dotted.
- Fig. 4. As fig. 3, but with the curves now corresponding to D-' and the 'hard pomeron' curves computed in ref.[2](dotted).

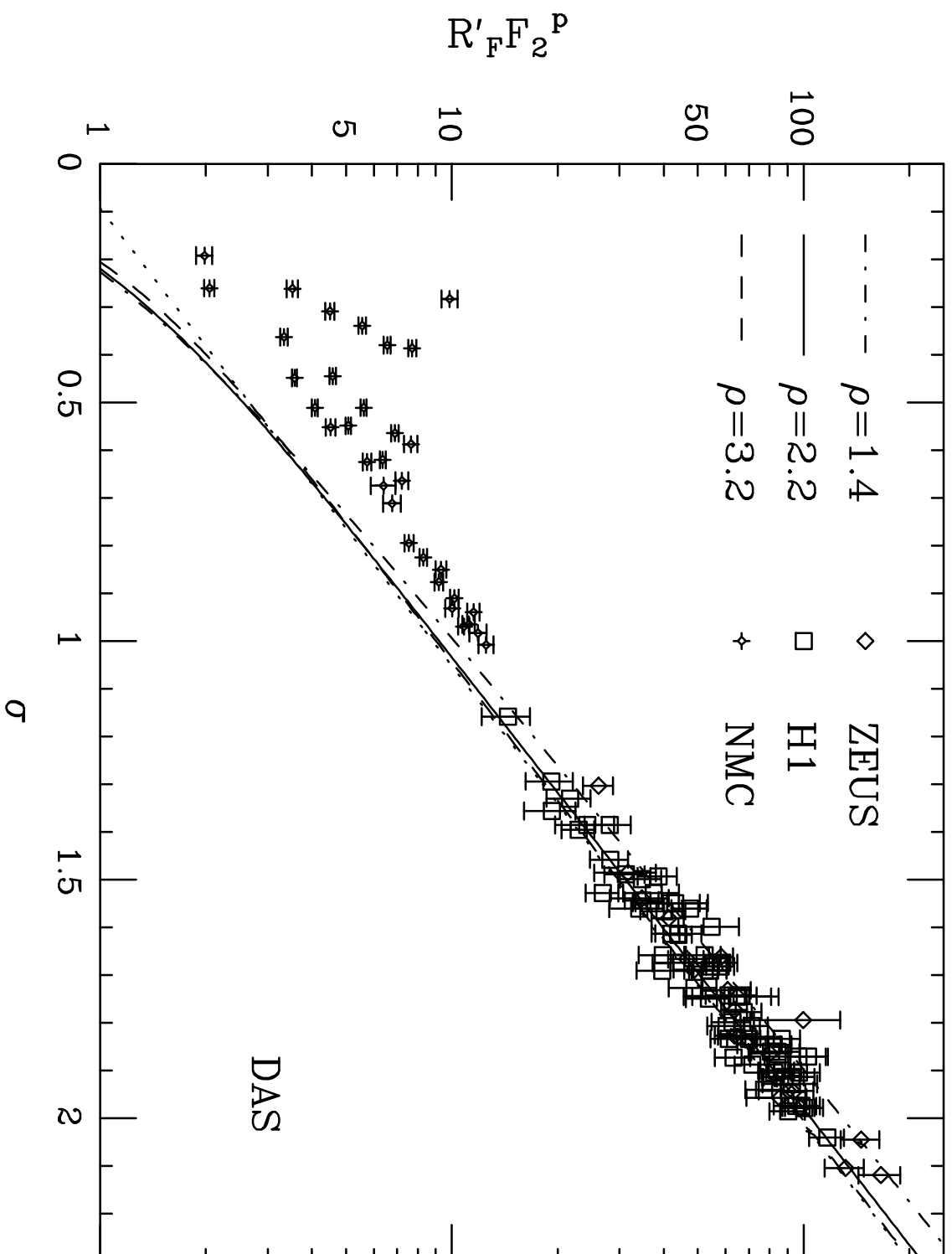


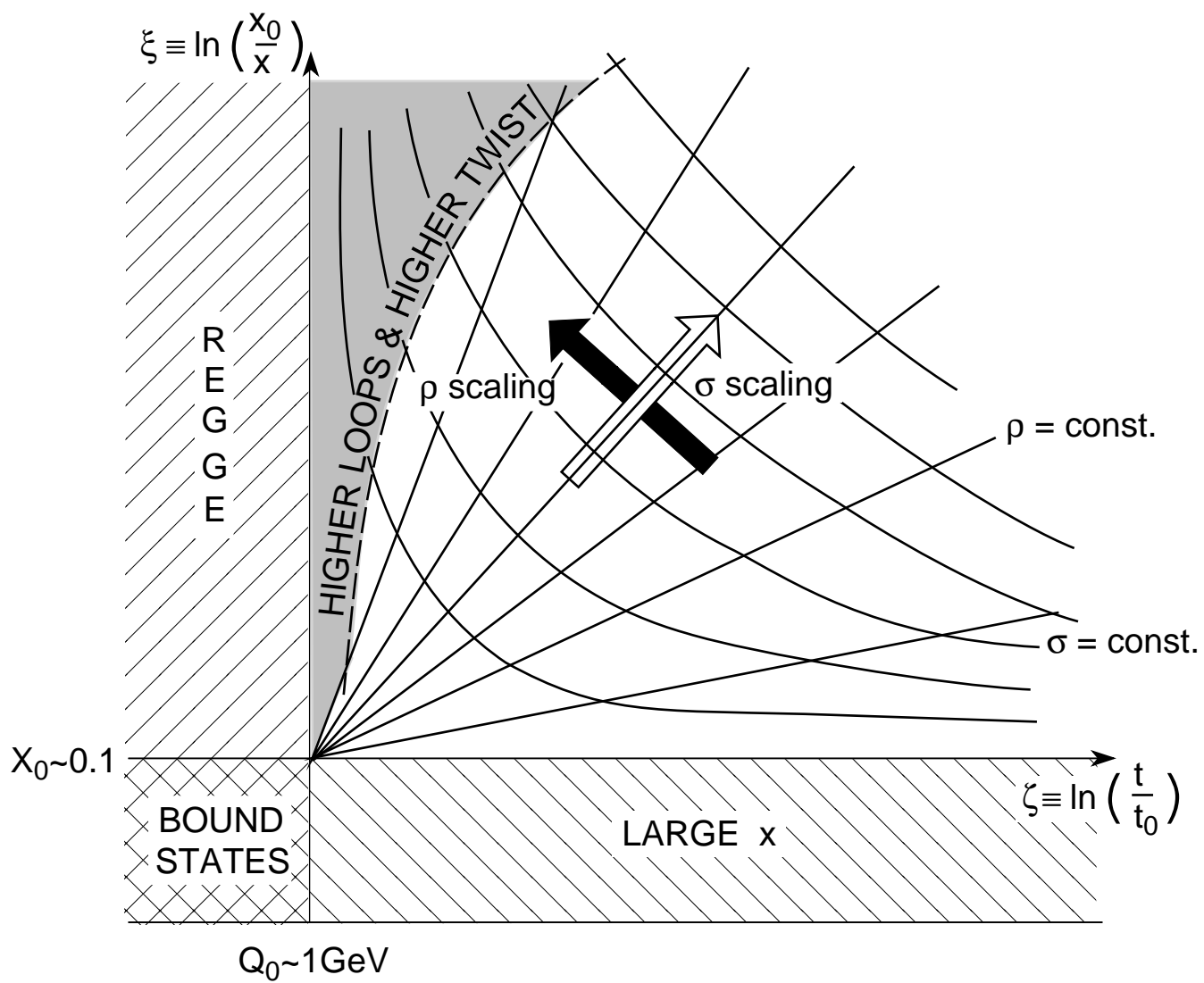
Fig. 1

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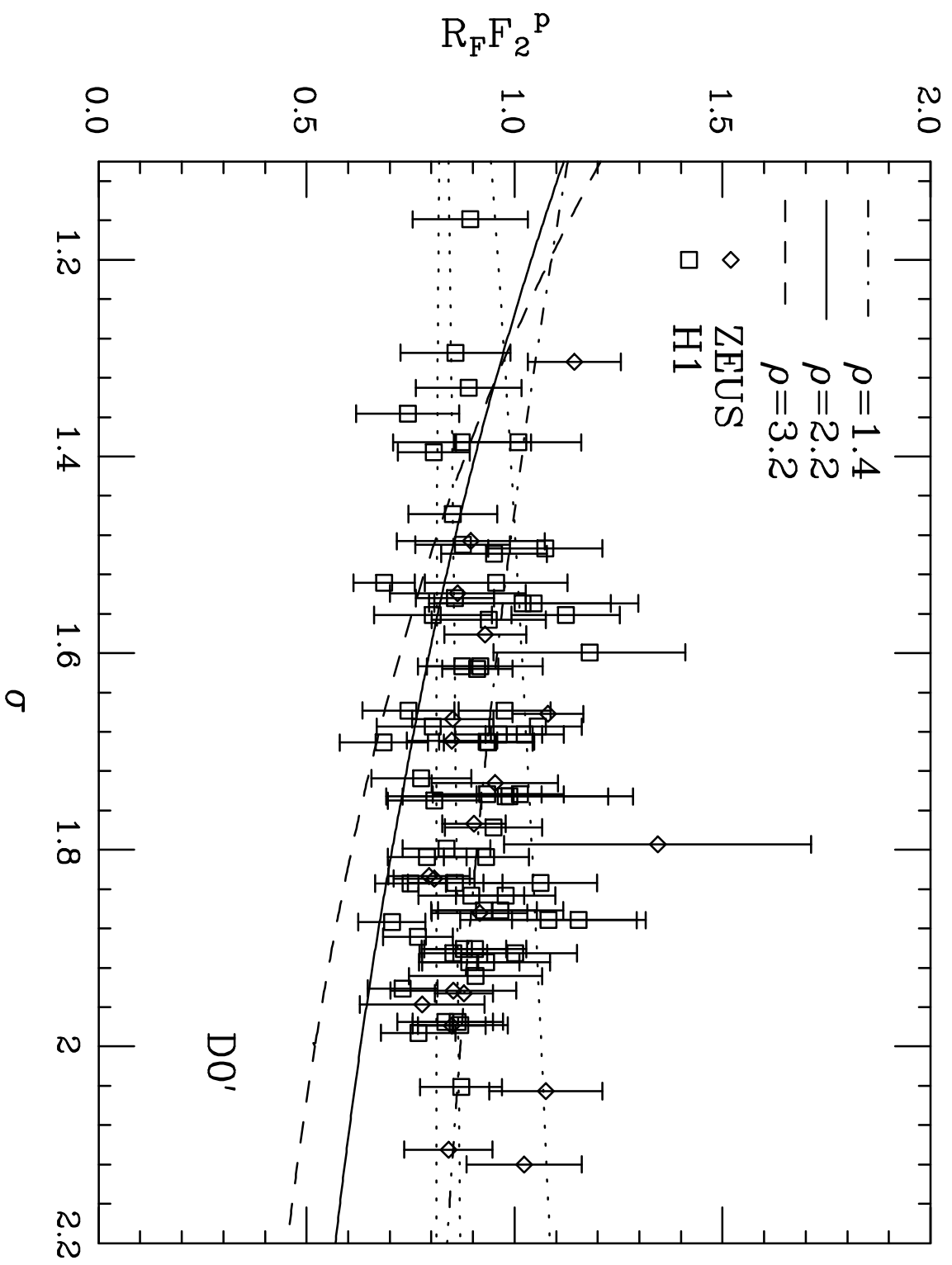


Fig. 3a

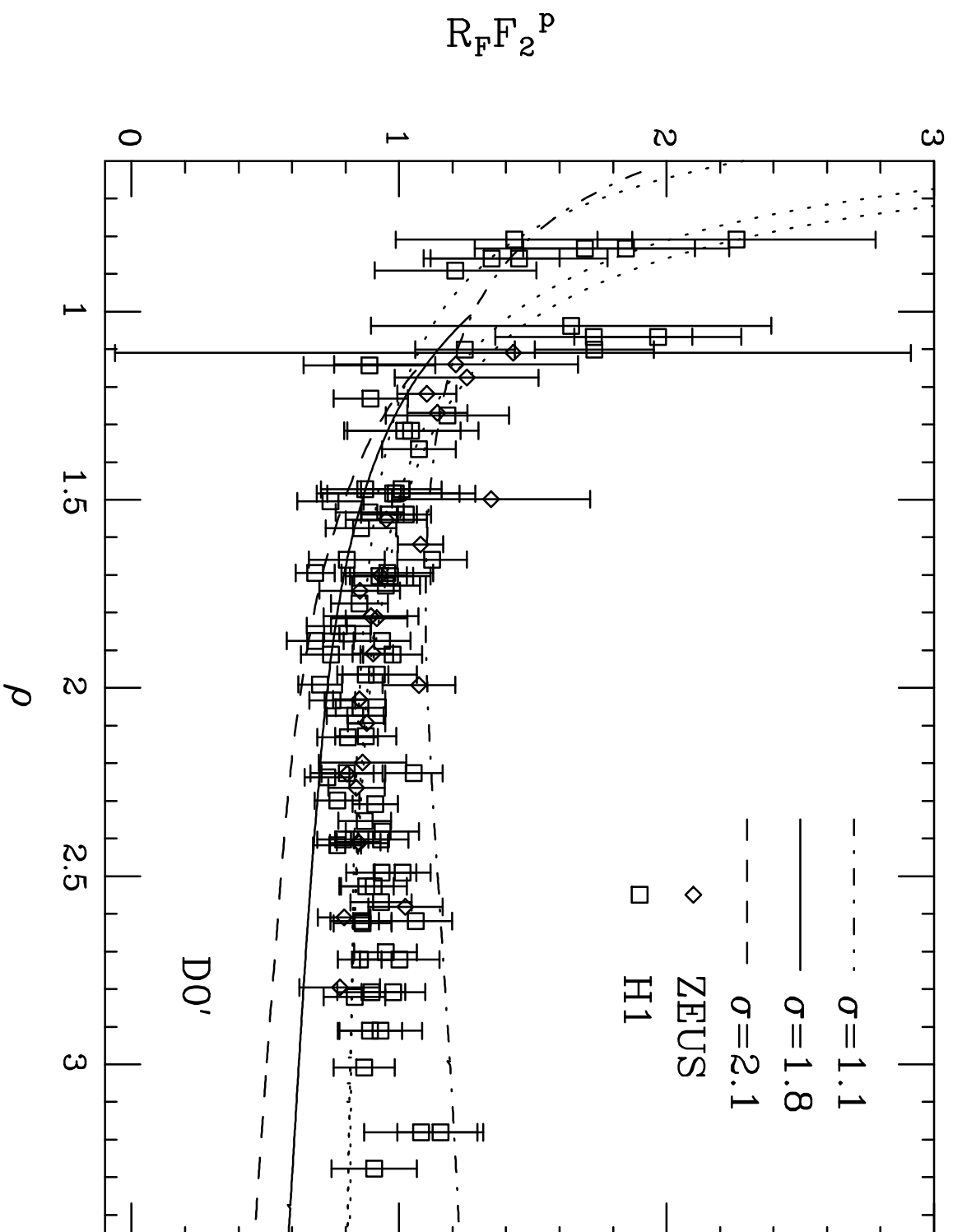


Fig. 3b

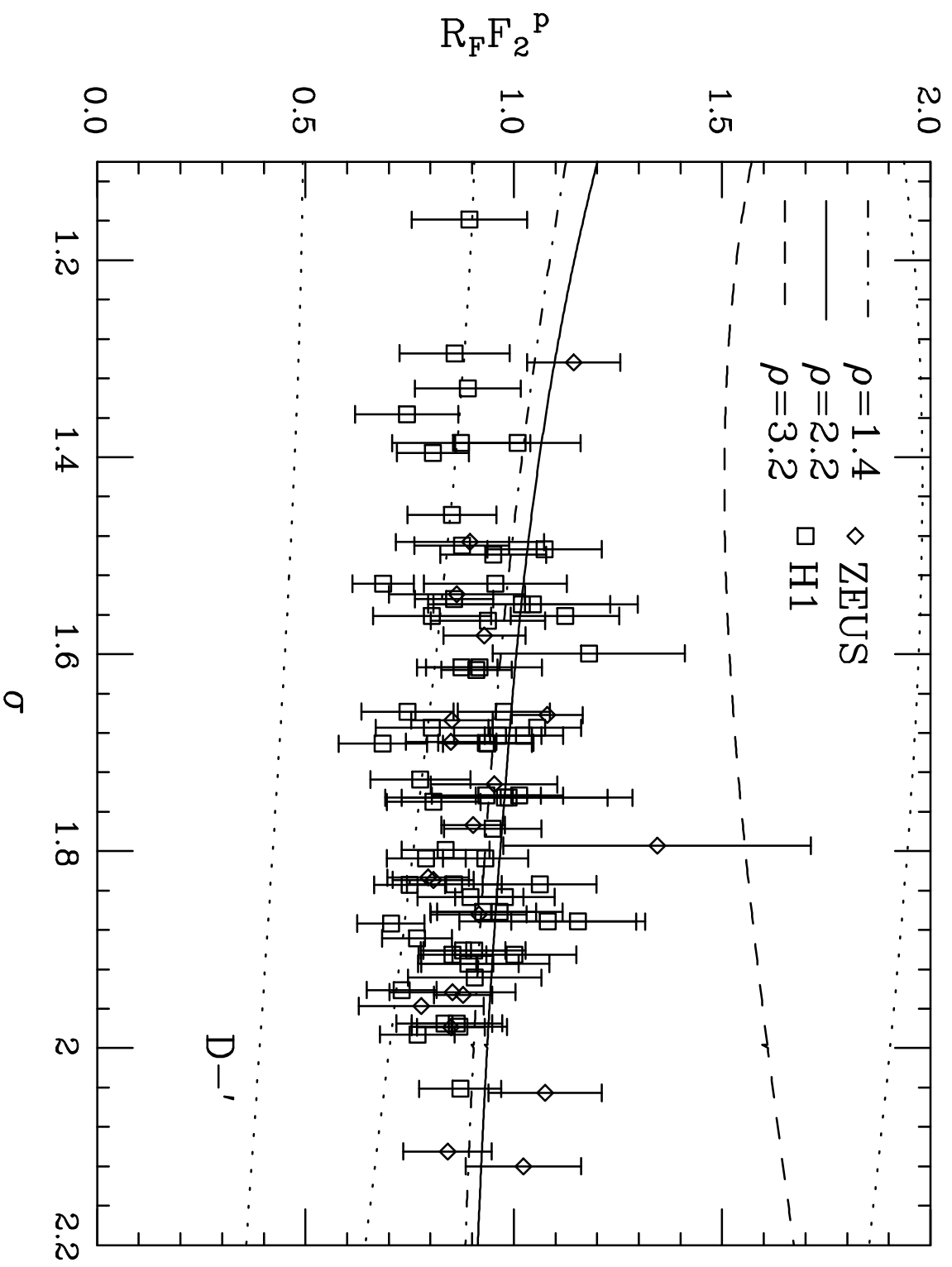


Fig. 4a

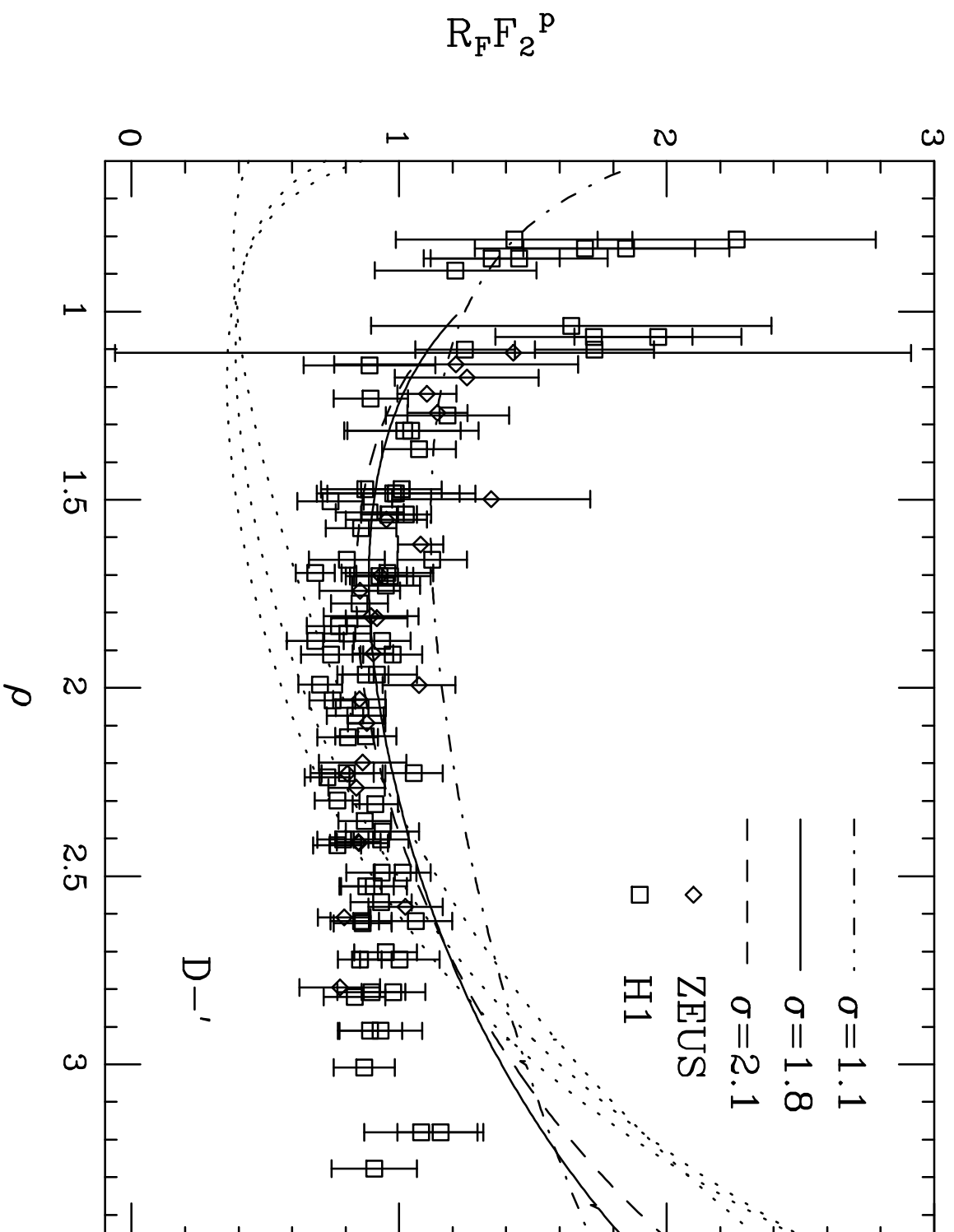


Fig. 4b